

AR Number: AR-001-500



DEPARTMENT OF DEFENCE DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION R.A.N. RESEARCH LABORATORY EDGECLIFF, N.S.W.

RANRL-TECHNICAL NOTE, No. 3/79

C COMMONWEALTH OF AUSTRALIA 1979

RESULTS OF ENVIRONMENTAL MEASUREMENTS ON THE NORTH-WEST SHELF AND EASTERN INDIAN OCEAN

(APRIL - MAY 1979)

JAJ H. L. MULHEARN



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SUMMARY

Oceanographic and meteorological results from RANRL Cruise 1/79 to the east Indian Ocean are presented. The oceanographic data for the Australian North-West Shelf show that over the shelf and slope there were both surface and bottom well-mixed layers, separated by a sharp thermocline and that in the vicinity of Rowley Shoals there was considerable variability in temperature profiles. Also in this region many of the temperature profiles off the shelf had a step-like structure. There appeared to be a southward current over the continental slope.

The meterological results often revealed the presence of small inversions at a height of approximately 100 m above the sea and showed that a sea-breeze at the coast was not felt 80 km or more out to sea.

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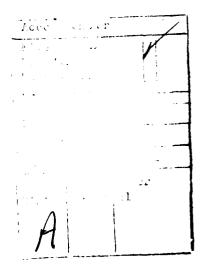
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DOCUMENT CONTROL DATA SHEET

Sec	urity classification of this page		UNCLASSIFIED	
1.	DOCUMENT NUMBERS:	2.	SECURITY CLASSIFICATION:	
	a. AR Numbers: 001-500 b. Document Series: RANRL Technical Note c. Report Number: 3/79		 a. Complete document: UNCLASSIFIED b. Title in isolation: UNCLASSIFIED c. Summary in Isolation: UNCLASSIFIED 	
3.	TITLE: RESULTS OF ENVIRONMENTAL MEA EASTERN INDIAN OCEAN (APRIL		EMENTS ON THE NORTH-WEST SHELF AND	
4.	PERSONAL AUTHOR(S):	5.	DOCUMENT DATA: November 1979	
	Mulhearn, Phillip J.			
		6.	TYPE OF REPORT AND PERIOD COVERED: Data Report; April - May 1979	
7.	CORPORATE AUTHOR(S):	8.	REFERENCE NUMBERS:	
	RAN Research Laboratory P.O. Box 706 Darlinghurst. NSW 2010		a. Task: NAV Task 76/59 b. Sponsoring Agency: NSA c. Approved	
		9.	COST CODE:	
10.	IMPRINT (Publishing estab- lishment):	11.	COMPUTER PROGRAM(S) (Title(s) and language(s))	
	RAN Research Laboratory		NIL	
12.	RELEASE LIMITATIONS (of the docume	nt i	in addition to the Distribution List)	
	AUSTRALIA: Approved for Public Re	leas	e.	
	OVERSEAS: N.O. P.R.	A	B C D E	
13.	ANNOUNCEMENT LIMITATIONS (of the			
14.	DESCRIPTORS:	atio 15.		
17.	Oceanography	13.	0810	
	Meteorology		0402	
16.	the east Indian Ocean are presented. The oceanographic data for the Australian North-West Shelf show that over the shelf and slope there were both surface and bottom well-mixed layers, separated by a sharp thermocline and that in the vicinity of Rowley Shoals there was considerable variability in temperature profiles. Also in this region many of the temperature profiles off the shelf had a step-like structure. There appeared to be a southward current over the continental slope. The meteorological results often revealed the presence of small inversions at a height of approximately 100m above the sea and showed that a sea-breeze at the coast was not felt 80 km or more out to sea. Approved for Public Release.			
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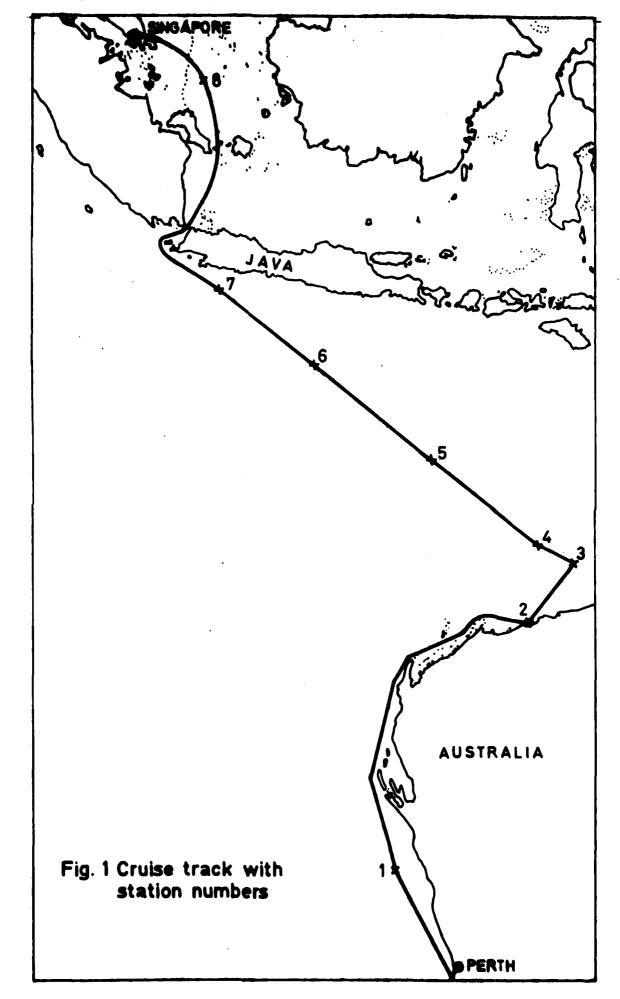


TABLE 1: OCEANOGRAPHIC STATIONS ON LEG 1

Station No	Position	Date	Time	Measurements Made
1	28° 40°s 113° 35°E	27/4/79	0900H to 1215H	Niskin cast (fluorescence salinity) T'meter XBT
3	18 ⁰ 25 ° S 120 02° E	30/4/79	0920Н to 2 345Н	2 Niskin casts (fluorescence, salinity) Timeter XBT, shipis set
4	17° 27.1'S 118° 55.8'E	1/5/79	1000H to 1 290E	Niskin (fluorescence salinity) T'meter XBT, ship's set
6	10 ⁰ 50°S 110 ⁰ 30°E	3/5/79	1230H to 1300H	Niskin cast (fluorescence salinity) XBT
7	08° 08°S 106° 42° E	4/5/79	1000GH to 1215GH	Niskin cast (fluorescence salinity) XBT, T [*] meter

Underway:

XBT Section from Station 3 to Sunda St

T'meter = Transmissometer

1. Introduction

RANRL Experiment 1/79 was a marine environmental cruise whose primary aim was to take measurements for the Oceanographic Programme of the Global Weather Experiment in Special Observory Period II (10 May-8 Jun 1979). These measurements were to be performed between 5°S and 5°N at 92°E in the Indian Ocean, from HMAS Diamantina.

Leg 1 of RANRL Experiment 1/79 which occupied the transit from Cockburn Sound, W.A. to Singapore had different aims. The main ones were (1) to obtain atmospheric temperature and humidity profiles at low levels over the sea under conditions appropriate for the formation of advective radar ducts (See Mulhearn, 1978) and (2) to obtain data on oceanic microbubbles, which are important for sonar propagation. The area chosen for this work was off the north— west Australian coast between Port Hedland and Broome.

Due to serious refuelling problems Leg 1 had to be greatly curtailed and within the time available atmospheric conditions were never suitable for advective radar duct formation. The profiles obtained will still be presented here and compared with atmospheric data obtained simultaneously at Broome and Port Hedland. The number of bubbles present was below the measurement threshold of the instrumentation so that in a sense the bubble experiments were unsuccessful. However, one can work out the maximum possible number of bubbles present and this will be the subject of a separate report.

Despite the problems a number of useful oceanographic measurements were obtained, especially in the North-West Shelf Area and the ones to be presented here are summarised in Table 1. They consist of fluorescence and salinity data from shallow Niskin casts at five stations, Transmissometer profiles at four stations, and some detailed XBT sections. These measurements were intended to complement the microbubble data.

2. Oceanographic Measurements

2.1 Cruise Summary

The ship departed HMAS Stirling at 1400 on 26/4/79. Figure 1 shows the cruise track and positions of stations occupied. Only oceanographic

data from stations 1,3,4,6 and 7 are presented here. An XBT section from station 3 to the Sunda Straits, between Java and Sumatra, was obtained with more detailed measurements being obtained for the southern end of that transit, especially over the continental shelf and slope. More details can be obtained from Cruise Leader's Report 1/79. (Mulhearn (1979).

2.2 Experimental Techniques and Procedures

Hydrological data were obtained from casts with Niskin bottles using HMAS DIAMANTINA's oceanographic winch. No reversing thermometers were used and depth of sampling bottles was inferred from the length of wire deployed measured via a metre-block and the wire angle at the surface. A 56 lb (25.4Kg) weight was attached to the end of the hydrographic wire. On those casts for which the wire angle was less than approximately 10° no wire angle was recorded. Casts were obtained with the ship drifting, no attempt being made to manoevre the ship on station so as to keep the hydrographic wire vertical at the surface.

Water samples were analysed for fluorescence, using a Model 111-004

Turner fluorometer in the cuvette mode, and for salinity, using an Inductive

Salinometer (Brown and Hamon, 1961).

At each station the fluorometer was zeroed using a filtered sample of surface water - the filtration being done through millipore filter papers. For later calibration a bulk sample from near the depth of maximum fluorescence was filtered with Mg CO₃ on the filter paper to neutralise any acidity on the sample. The filter paper was then placed in an open tube in a vacuum dessicator and the dessicator containing all filter papers obtained to date, stored in a fridge (not frozen). All samples were left in the dessicator in the fridge till 21 May 1979, when the glass tubes containing the filter papers were sealed with plastic stoppers and stored in wide mouthed thermos flasks for transport, but were left in the fridge by mistake. The thermos flasks were taken out of the fridge on the 18th June 1979, by CSIRO personnel and brought back to Sydney on 10 June 1979, where they were left in refrigeration awaiting chlorophyl analysis.

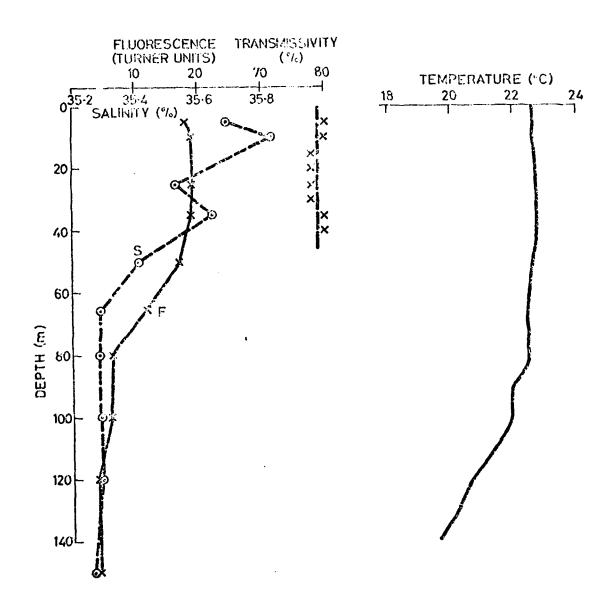


Fig. 2 Casts at 28°40'S, 113°35'E on 27/4/79.

Salinity analysis was done on board by the CSIRO experiment team during Leg 3. Transmissometer measurements were taken with an Oceanographic Engineering Corp Transmissometer Model 410-BR using standard setting up procedures as set out in the instrument's handbook. All temperature profiles were obtained with standard Plessey Sippican 450 XBT probes and recording system.

2.3 Results

Station 1: This was a trial run en route between Cockburn Sound and the North-West Shelf area, and results from it are presented here, in Figure 2, only for archival purposes.

Measurements in the North-West Shelf Area

Positions of XBT drops and Niskin casts are shown on Figure 3, and XBT times and positions are shown in Table 2. XBTs were numbered consecutively from the start of the cruise. The ship was drifting from 0920H to 2345H on 30th April, within which time XBTs 4 to 8 were dropped and Niskin casts made at 1030H and 2000H. The ship then steamed to a position near where XBT number 11 was dropped. XBTs 11-14 were dropped while the ship drifted from 1000H to 1200H on 1st May and a Niskin cast was made at 1000H. The ship then steamed off along the track shown dropping XBTs hourly up till XBT Number 23 and 4-hourly thereafter, till Sunda Strait.

Isotherms from the continental shelf region out to XBT number 19's position are presented in Figure 4. The bottom contour is that inferred from the ship's shallow echo-sounder. (It is compared with that from the chart for this region, drawn up in 1965, in Figure 5. The values off the chart were estimated from sounding near to the ship's track.) Over the continental shelf and slope, bottom well-mixed layers were observed, that measured by the XBT Number 10 being approximately 100m deep. Surface currents between 0.5 and 1.3 knots, probably tidal, were observed while the ship was drifting at the positions of XBTs 4 to 8 and a 0.75 knot current was observed near Rowley Shoals (XBT 11 to 14). (See Table 3). The observed bottom well-mixed layers would be caused by these strong currents.

TABLE 2

XBT No	Depth	Lat	Long
4 56 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	110M "1 120 110 160 340 450 390 "1 402 500 520 1300 1400 2112 2790 5283	18 25 18 25 18 27 18 33 18 12 17 47 17 27 17 27 17 27 17 25 17 25 17 26 17 57 16 36	120 01 120 00 119 56 119 56 119 44 119 56 118 56 118 56 118 56 118 40 118 40 118 12 117 57 117 45
22 2 3	5653 5739	16 2 6 16 19	117 34 117 20

TABLE 3 - SHIPS' SET VECTORS FROM SHIP DRIFTING

XBT Nos	Ships Drift	Notes
5 to 6 6 to 7 7 to 8 11 to 14	0.5 knots at 200° 1.2 knots at 280° 1.3 knots at 155° 0.75 knots at 103°	from Sat Nav fixes """"""" from bearing & range to Cunningham Is. Rowley Shoals

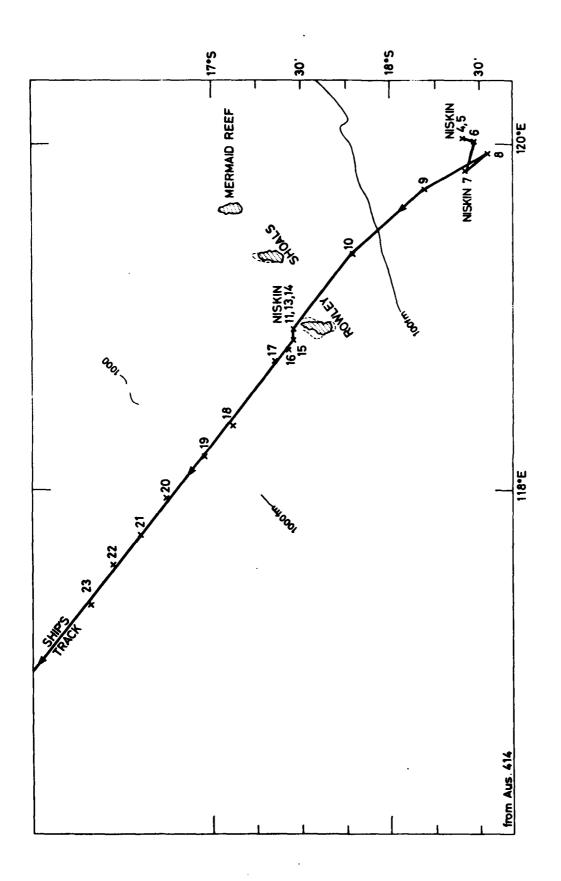


Fig. 3 Positions of XBT and Niskin casts and ship's track, on N.W. shelf. (Numbers referto XBT numbers.)

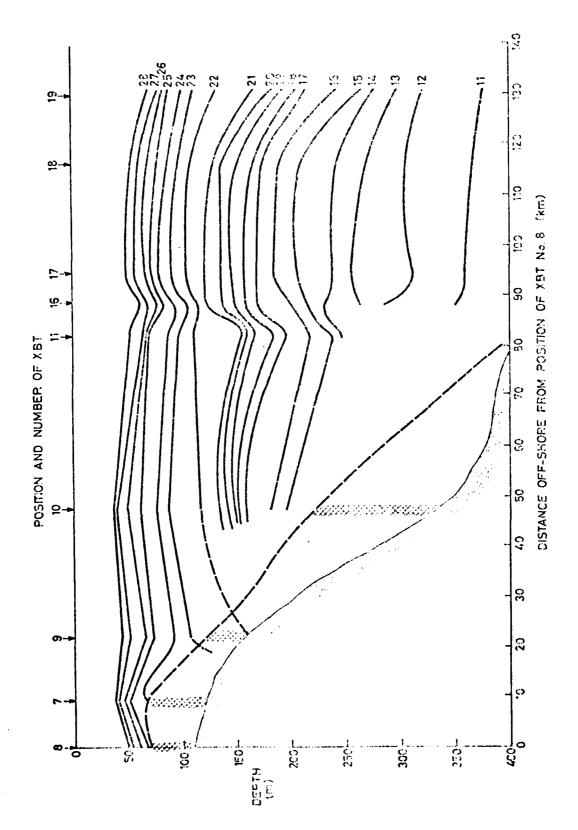


Fig. 4 XBT cross section in continental slope region off N.W. coast. Hatching above bottom denoted well mixed layers.

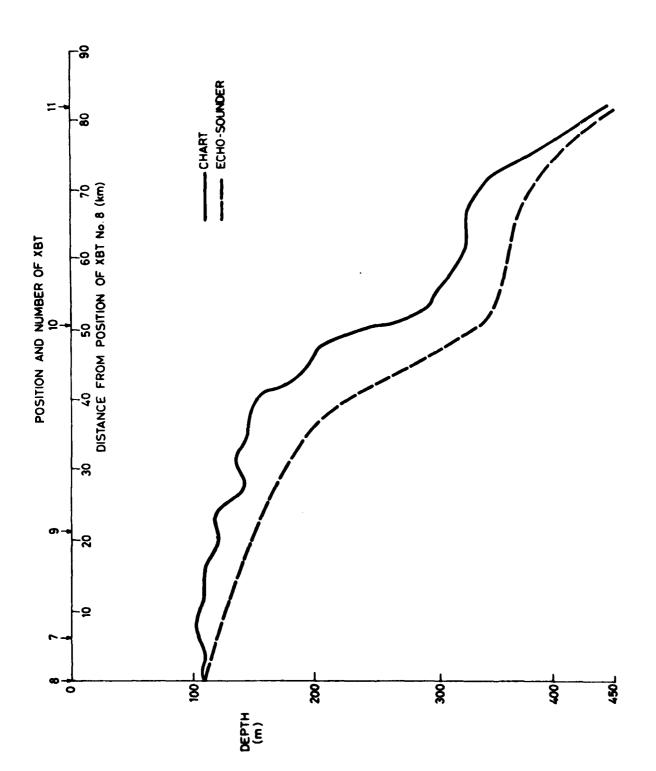


Fig. 5 Bottom profile along XBT section of Fig. 4 from chart and ship's echo-sounder.

In Mgure 5, ship's echo-sounder results have not been corrected for tide height. Diamantina's position was estimated from dead-reckoning between single Sat-Nav fixes, and could be in error.

Over the continential slope region, out to the position of XBT Number 10, the isotherms slope upwards with distance off-shore, indicating a southwards flowing current. This current was even more evident 3 weeks later when a CSIRO Cruise (Courageous Cruise 49) traversed the same area. Seawards of XBT 10's position the isotherms tend to slope downwards with distance off-shore, indicating a northwards flow. A sharp front is clearly evident between XBTs 11 and 16. This discontinuity is close to the southenmost of the Rowley Shoals and is most likely a local effect due to this reef.

Individual XBT's are shown in more detail in Figures 6(a)-(c). XBT's 7 to 10 in Fig 6(a) show well-mixed bottom layers, and step-like structure is observable in XBT's 10 to 19, throughout the lepth of the profiles. XBT's 13 and 14 in Fig 6(b), were taken at the station near Rowley Shoals 1½ hours apart while the ship was drifting at 0.65 Knots, in direction 103°. All the XBT's in Fig 6(b) were taken close together and show a high degree of variability, which may be the result of flow around the reef. Note that the flow direction may vary markedly with depth.

Niskin casts were obtained over the shelf at 1030 local time near XBT No. 5's position and at 2000 near XBT No. 7's position. On figure 7 the fluorescence and salinity profiles obtained are presented along with XBT traces and a Transmissometer profile obtained in the same area. The well-mixed surface layer for salinity appears to be 13m deeper than that for temperature, but this could be due to an error in the estimates of Niskir bottle depths. It can be seen that fluorescence peaks within the thermocline, just below the mixed layer. Fluorescence is a measure of the quantity of phytoplankton present and is thus perhaps related to rates of bubble production.

And ther Niskin east was obtained at the Rowley Shoals station (See Fig 8). There is little variation in salinity with depth[†] and the maximum in fluorescence is broader. Transmissivity is higher here indicating that water clarity is increasing with distance offshore, as one would expect.

⁺ The high salinity value of 35.3% at 60m depth is assumed to be erroneous.

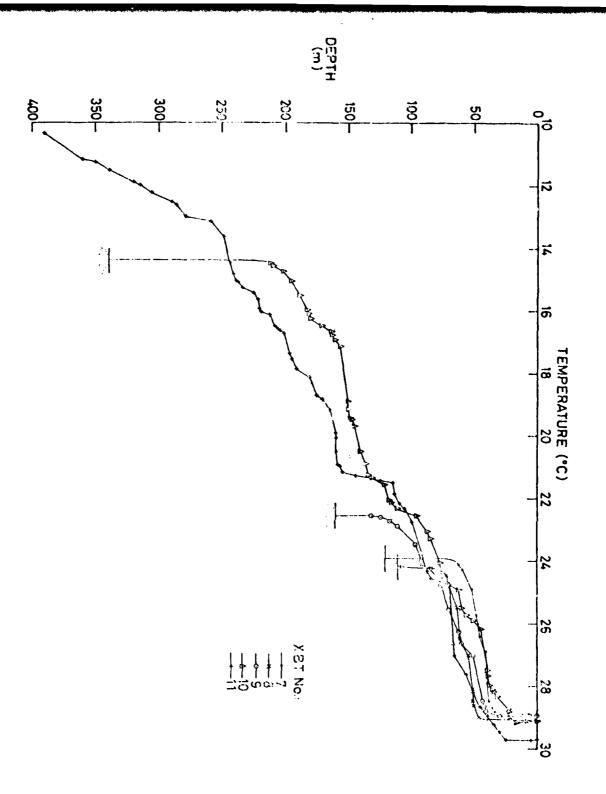


Fig. 6 (a) XBT traces 7 to 11 over continental shelf and slope.

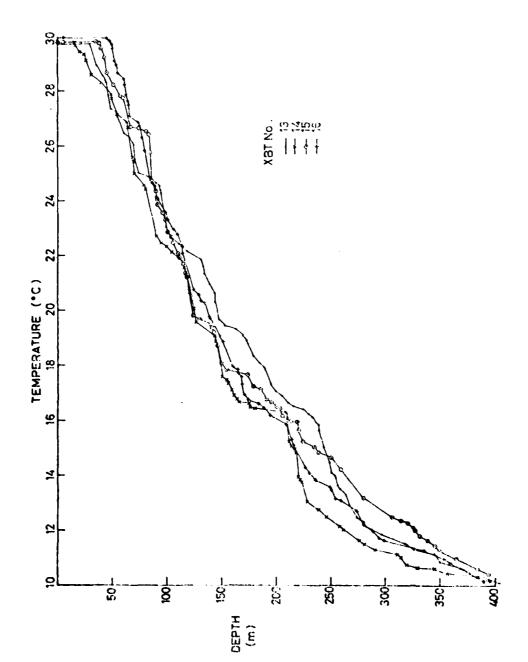


Fig. 6 (b) XBT traces 13 to 16 over continental slope

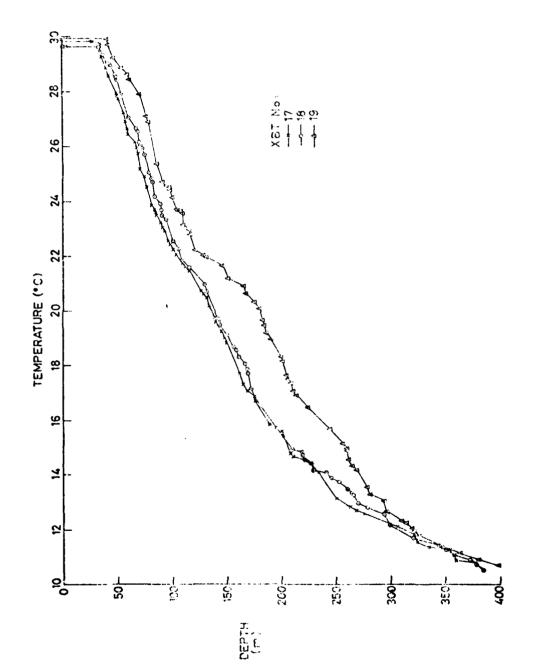


Fig.6(c) XBT traces 17 to 19 over continental shelf.

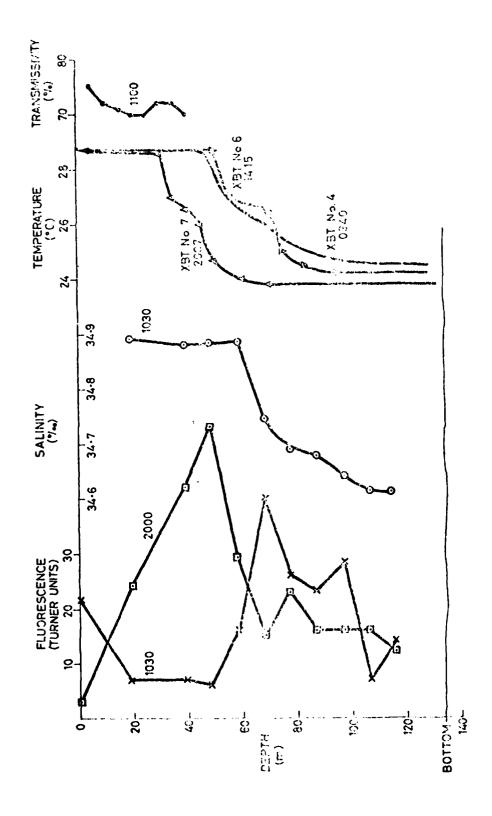


Fig. 7 Casts over N.W. shelf on 30/4/79. Numbers on curves denote local time (H) of measurements. 0940H cast was at 18°25'S, 120°02'E, 2005 cast was at 18°25'S, 119°48'E.



Fig. 8 Cast at 17°27.1'S, 118°35.8'E on 1/5/79.

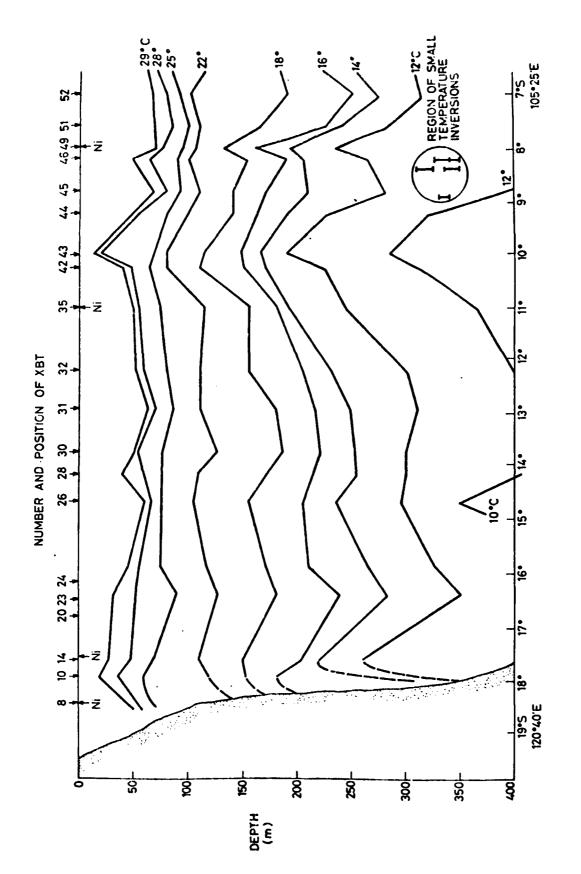


Fig. 9 XBT section from N.W. shelf to Sunda Strait. (Ni denotes Niskin cast position.)

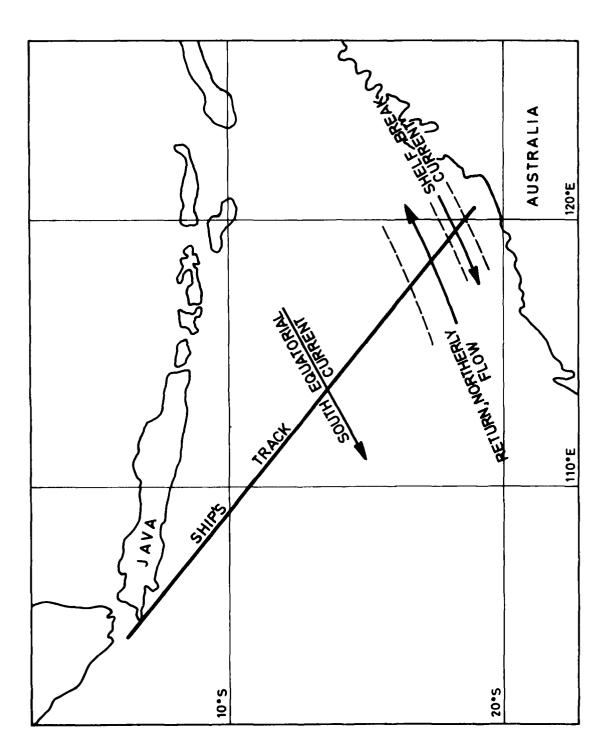


Fig. 10 Inferred current directions from XBI section.

Measurements Across to Sunda Strait

The XBT section from the North-West shelf Area across to Sunda Strait is shown in Figure 9. As in Figure 4 the isotherms slope up with distance off shore, near the Australian coast, indicating a southwards current. Further out the isotherms slope down indicating a northwards flow but beyond XBT Number 23's position (about 16°S) the deeper isotherms slope up till about 10½°S indicating that the current perpendicular to the ship's track is towards the south-west. This is probably the South Equatorial Current. North of 10°S the isotherms slope down towards the coast of Java with a pronounced peak near 8°S, indicating, perhaps a flow, with either eddies or meanders, to the East. Inferred current directions are shown on Figure 10. This XBT Section is different from those of Speechley (1977) which show very light slopes to the isotherms between Christmas Island and Sunda Strait.

Niskin casts were taken at Stations 6 and 7 en route to Sunda Strait and these are shown along with XBTs in Figures 11 and 12 respectively. Note the temperature inversion in the XBT trace in Figure 12, which was found in several traces in this region, just south of Java. Note also the marked difference in the salinity profiles at these two locations.

2.4 Discussion

The most interesting oceanographic results were those obtained in the North-West Shelf area. The temperature profiles showed a high degree of variability in space and time, especially near Rowley Shoals, and there was a step-like structure in the temperature profiles which appeared to become less pronounced with distance off-shore. Another marked feature was the depth of the bottom well-mixed layers. From the XBT profiles a southward current was inferred to be present over the shelf-break, but this was much more pronounced several weeks later in the course of a CSIRO cruise in this region.

A similar decrease of step-like structure with distance off-shore can be seen in the data of Godfrey and others (1980) from the eastern end of Bass Strait.

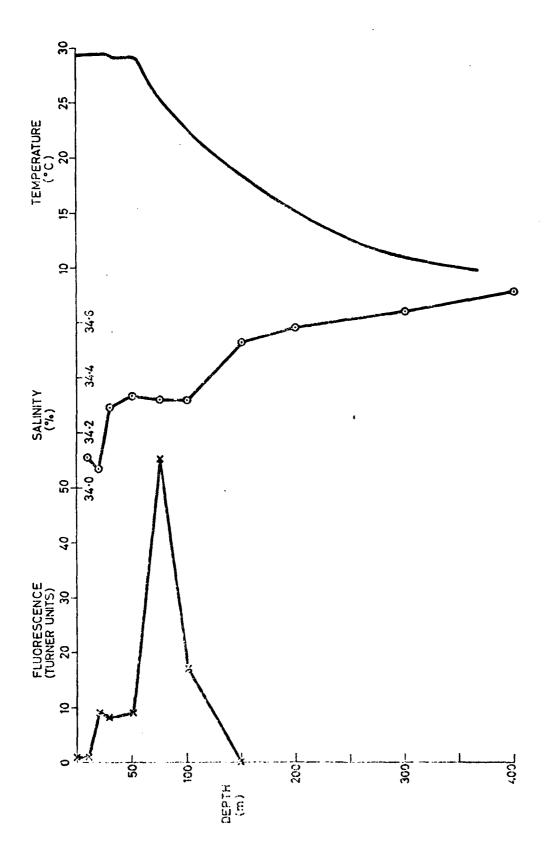


Fig. 11 Cast at 10°50'S, 110°30'E on 3/5/79.

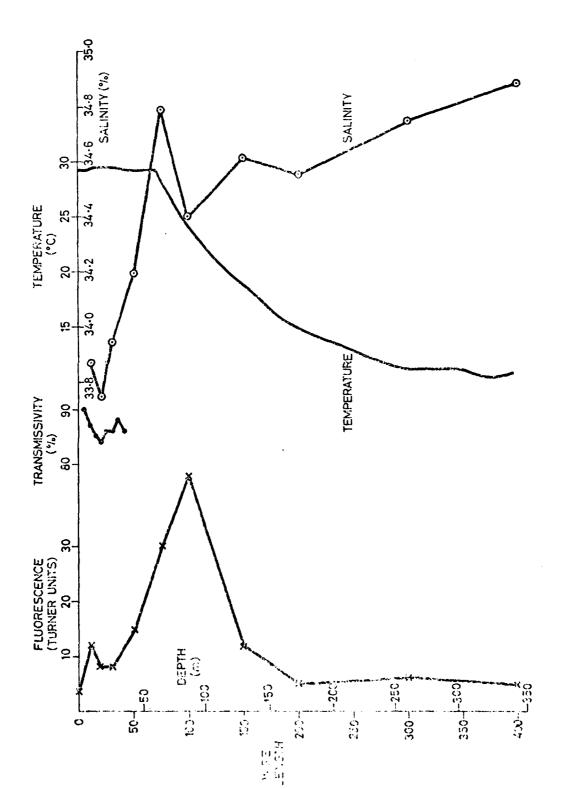


Fig. 12 Cast at 08°08'S, 106°42'E on 4/ /79.

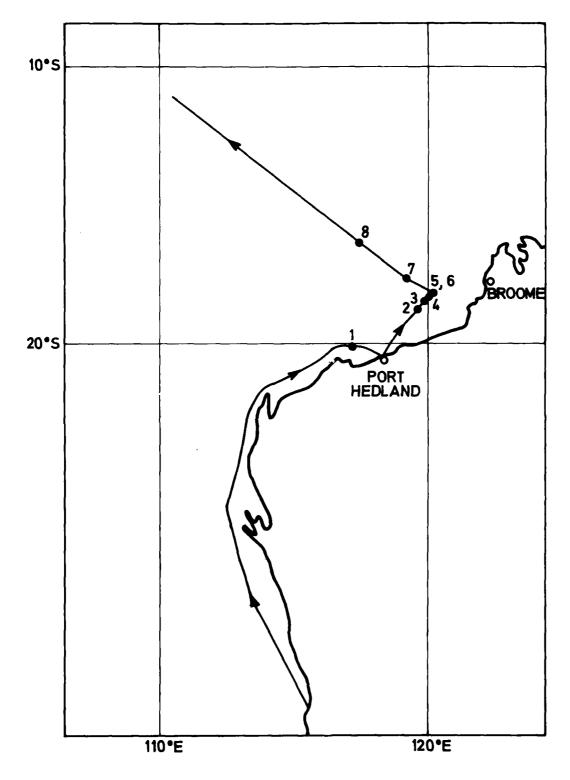


Fig. 13 Positions of atmospheric soundings No. 1 to 8.

3. Atmospheric Measurements

Atmospheric soundings were taken at the locations shown on Fig 13 on 29 April to 1 May 1979. The results can be examined for the modifying influence of the sea as an air mass passes over it from the land. However, because the winds over the sea were light and variable, few conclusions could be reached. Also there was often a sea-breeze at the coast with an off-shore flow aloft which further complicated interpretation. No influence of the sea-breeze was felt at sea at locations 80km to 180km off-shore.

3.1 Experimental Method

The Bureau of Meteorology made available the following records from Broome and Port Hedland:-

Radiosonde flights

Radar winds

Surface recordings of pressure, temperature, wet bulb temperature, wind-speed and direction, cloud and significant weather events.

On the ship, some soundings were made with the Standard Met. Bureau radiosonde equipment. Some were made with a modified "temperature only" sonde
to provide a continuous temperature trace, while others were made with sondes
containing two humidity sensors rather than the normal temperature and humidity
elements so as to obtain a continuous humidity signal. These continuous
humidity soundings require a temperature sounding to be made as closely as
possible because the humidity sensors' outputs are afunction of temperature.
The height of the temperature sonde as a function of time is found from a
calculation of rise-rate (see appendix)

3.2 Stations

Soundings were made at the locations shown on Figure 13. The dates and times of soundings and some relevent meteorclogical information is shown in Table 4.

3.3 Results

29 April - The humidity only sonde was not accompanied by a temperature sonde so that humidity values could not be extracted from it. It revealed a steady humidity decrease from the surface to about 100m then an increase for

id Notes		Sea Breeze at Coast	Sea Breeze at Coast
Winds on Land	70°/4kt	120 °/? 7 0°/ 8 170°/8 30°/8 0°/6	130 <mark>0</mark> /? 272 ⁰ /?
Ship Winds	130 <mark>0</mark> /15kt	120 <mark>°</mark> /15 90°/11 100°/10 100°/9	Calm 60°/10
Type	Humidity only	Radiosomde Humidity only Temp " Humidity " Temp "	Radiosonde "
Time	1052Н	0715H 1114H 1330H 1530H 1600H	0715H 1915H
No	-	0 W 4 1V 0	8 2
Date	4/67	30/4	1/5

*Shore winds are at Port Hedland for 29/4/79 but at Broome for 30/4/79 and 1/5/79

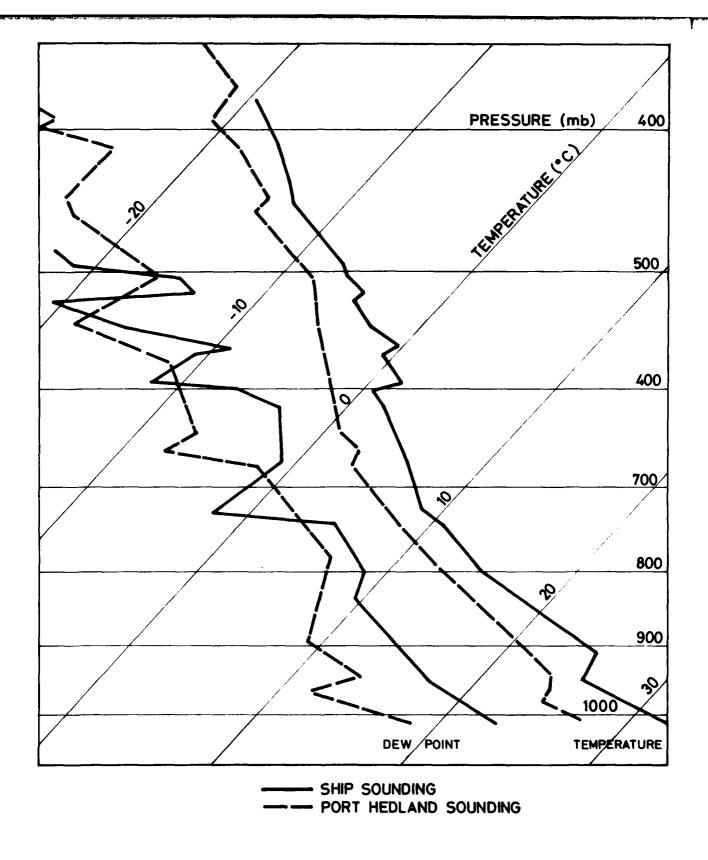


Fig. 14 Radiosonde flights at 0715H on 29/4/79.

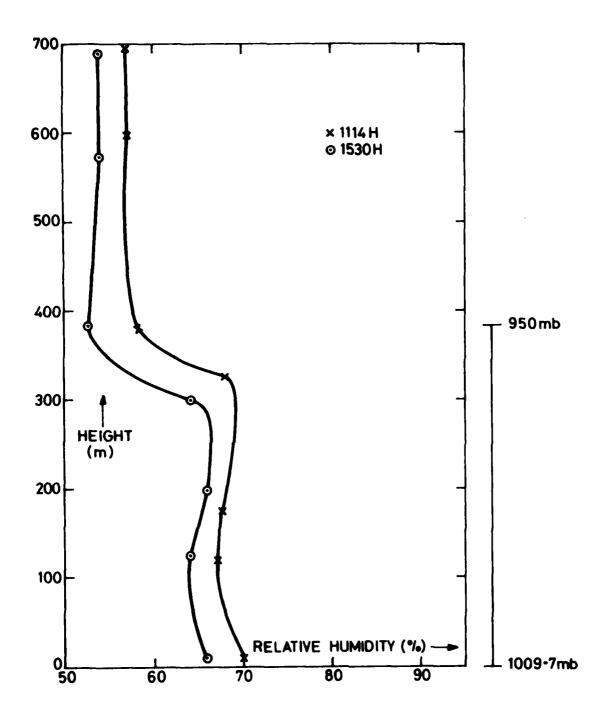


Fig. 15 Humidity soundings on 30/4/79 from ship.

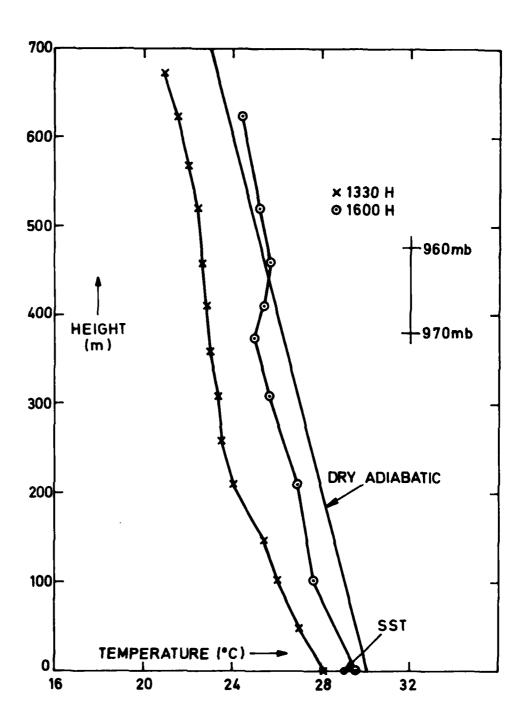


Fig. 16 Temperature soundings from ship on 30/4/79.

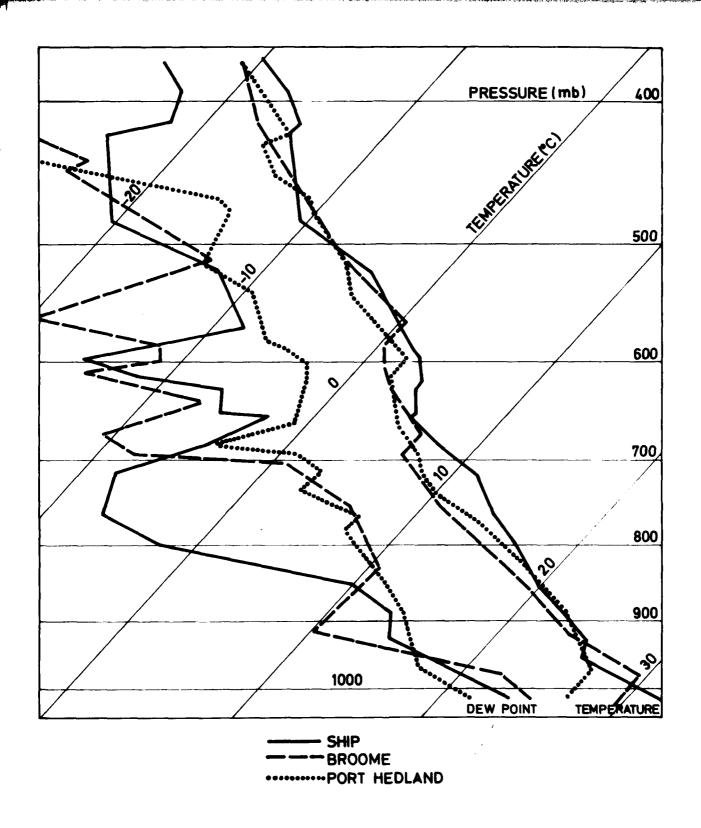


Fig. 17 Radiosonde flights at 0715H on 1/5/79.

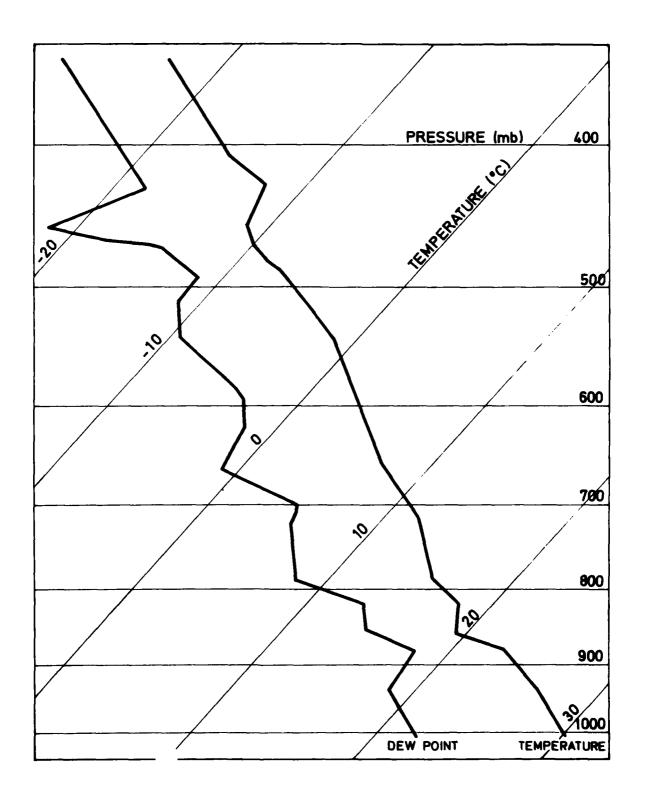


Fig. 18 1915H radiosonde flight from ship on 1/5/79.

approximately the next 50m with a further decrease above that. The seasurface temperature was 28.3°C and air temperature was 24.4°C. The wind was off the land, which was about 80km upwind. The reason for the slight inversion is unknown.

30 April - Radiosonde flights from the ship and from Broome at 0715H are shown on Figure 14. The temperature trace over land appears to be of order 3°C cooler than that over the sea at all heights while over the sea the low level inversion between 950mb and 910mb appears more pronounced.

Humidity traces from the 1114H and 1530H soundings are shown in Figure 15, while temperature traces from the 1330H and 1600H soundings are shown in Figure 16. No inversion is apparent in the 1330H temperature trace but it can be seen in the others at approximately 400m to 500m in the 1600 temperature trace and at 300m to 400m in the humidity traces. The difference in heights may be due to errors in calculating the sondes' ascent rates. A principal source of error was the difficulty, with the available apparatus, of accurately measuring the balloon's lift.

1 May - Radiosonde flights from the ship and from Broome and Port Hedland for 0715H are shown in Figure 17. The ground-level inversion over land appears to have been eroded by passage of the air over the sea leading to a well-mixed region up to 950mb with a small inversion between 980 mb and 930mb. The 1915H radiosonde flight is shown on Figure 18. Because the wind was from 060° it is impossible to assess the influence of the land mass, if any.

3.4 Discussion

In many of these traces weak low-level inversions were observed. These may result from convective mixing of the air as it passes over a patch of water warmer than its surroundings. The low-level of these inversions makes it unlikely that they are the result of modification of the air as it moves from over the land to over the sea. These weak inversions would have no significant effect on radar propagation.

Although there were strong sea breezes at the coast, no effect of these was felt at sea 80 km to 180km off shore.

Acknowledgements

Thanks are due to the officers and crew of HMAS Diamantina and the members of the RANRL scientific team who all helped greatly in obtaining these results. Thanks are also due to Mr. T. Golding and Dr. K. Sainsbury of CSIRO Division of Fisheries and Oceanography for many helpful discussions on the North-West Shelf area and to Mr. B. Scott of the same institution for the use of his fluorometery equipment. I would also like to acknowledge Mr. R. Mackenzie of the Bureau of Meteorology who gave a great deal of assistance and advice on the use of radiosondes.

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Appendix: Balloon Ascent Rates

The buoyancy of a met. balloon plus attachments is balanced by its aerodynamic drag as it rises at a constant velocity V.

$$\text{Drag} = \frac{1}{2} \rho \text{a Cd AV}^2$$
, ----- (A1)

where $Q_a = air density$

Cd = drag coefficient = 1.0 for a sphere

A = Cross-sectional area of balloon $= \pi r^2$

and r = balloon radius.

When the balloon is inflated its lift at lift-off is measured as the sum of a weight pan Wp plus any additional weights added, Wa

So:
$$Drag = Wp + Wa - S$$
, ----- (A2)

where S is the weight of any attachments, which are added after the lift is measured. In the cases considered the only attachment is the sonde.

Lift of balloon without attachments =
$$\sqrt{a} - \frac{he}{he} = \frac{4}{3} \pi r^{3}$$

= Wp + Wa + B,

where B = weight of balloon

HE = density of helium

From eqns (A1) and (A2)

$$V^2 = \frac{Wp + Wa - S}{\frac{1}{2} / a \, CdA},$$

 $A = \mathcal{H}r^2$

From (A3)

$$\mathbf{r} = \left[(Wp + Wa + B) / \left\{ (A + HE) \frac{4}{3} \right\} \right]^{\frac{1}{3}},$$

and so

$$V^{2} = \frac{2(Wp + Wa - S)}{\rho_{aCd} \pi \left[(Wp + Wa + B) / (Pa - PHE) / 4/377 \right]^{\frac{2}{8}}}$$
Assuming ρ_{a} He and taking

$$V = \frac{3.906 \text{ (Wp + Wa - S)}^{\frac{1}{2}}}{\text{(Wp + Wa + B)}^{\frac{1}{3}}} = \frac{3.906 \text{ (A4)}}{\text{(Wp + Wa + B)}^{\frac{1}{3}}}$$

Equation (A4) was used for calculating balloon ascent rates in this report.